OD Covariance in Conjunction Assessment: Introduction and Issues

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Why You're Receiving this Briefing

- **Since inception, CARA has required owner/operator predicted ephemerides, at least for maneuverable satellites**
- **Around 2012, CARA began requesting predicted covariance as well**
  - If/as possible for existing missions
  - Included in ICD/OA for new missions
- **CARA recently received two actions**
  - MOWG Action Item 1309-05
  - ESMO Maneuver Process Review RFA-02
- **Primary objectives of actions**
  - Why is CARA asking for predicted covariance from o/o
  - Any implementation recommendations
- **This briefing is in response to those actions**
- **Right table indicates which ESC missions are currently providing predicted covariance to CARA**

Mission	Y/N
Aqua	N
Aura	N
CALIPSO	N
CloudSat	N
EO-1	N
GCOM-W1	N
Landsat-7	N
Landsat-8	Y
OCO-2	Y
Terra	N

Agenda

- Covariance basics
- Use of covariance in probability of collision (Pc) calculation
- Covariance generation and propagation methods
- Covariance tuning
- Covariance theory compatibility
- CARA O/O covariance needs
- Conclusions

OD Solutions

- **Purpose of OD**
  - Generate estimate of the object's state at a given time (called the *epoch time*)
  - Generate additional parameters and constructs to allow object's future states to be predicted (accomplished through orbit *propagation*)
  - Generate a statement of the estimation error, both at epoch and for any predicted state (usually accomplished by means of a *covariance matrix*)
- **Error types**
  - OD approaches (either batch or filter) presume that they solve for all significant systematic errors
  - Remaining solution error is thus presumed to be random (Gaussian) error
  - Sometimes this error can be intentionally inflated to try to improve the fidelity of the error modeling
  - Nonetheless, presumed to be Gaussian in form and unbiased

OD Parameters Generated by ASW Solutions

- **Solved for: State parameters**
  - Six parameters needed to determine 3-d state fully
  - Cartesian: three position and three velocity parameters in orthogonal system
  - Element: six orbital elements that describe the geometry of the orbit
- **Solved for: Non-conservative force parameters**
  - Ballistic coefficient (C_DA/m); describes vulnerability of spacecraft state to atmospheric drag
  - Solar radiation pressure (SRP) coefficient (C_RA/m); describes vulnerability of spacecraft state to visible light momentum from sun
- **Considered: ballistic coefficient and SRP consider parameter**
  - Not solved for but "considered" as part of the solution
  - Derived from information outside of the OD itself
  - Discussed later

OD Uncertainty Modeling

- **Characterizes the overall uncertainty of the OD epoch and/or propagated state**
  - Uncertainty of each estimated parameter and their interactions
- **This is a characterization of a multivariate statistical distribution**
- **In general, need the four cumulants to characterize the distribution**
  - Mean, variance, skewness, and kurtosis; and their mutual interactions
  - Requires higher-order tensors to do this for a multivariate distribution
- **Assumptions about error distribution can simplify situation substantially**
  - Presuming the solution is unbiased places the mean error values at zero
  - Presuming the error distribution is Gaussian eliminates the need for the third and fourth cumulants
  - Error distribution can thus be expressed by means of variances of each solved-for component and their cross-correlations
  - Thus, error can be fully represented by means of a covariance matrix

Covariance Matrix Construction: Symbolic Example

- Three estimated parameters (a, b, and c)
- Variances of each along diagonal
- Off-diagonal terms the product of two standard deviations and the correlation coefficient (ρ); matrix is symmetric

	a	b	c	…
a	σ_a^2	$\rho_{ab}\sigma_a\sigma_b$	$\rho_{ac}\sigma_a\sigma_c$	…
b	$\rho_{ab}\sigma_a\sigma_b$	σ_b^2	$\rho_{bc}\sigma_a\sigma_c$	…
c	$\rho_{ac}\sigma_a\sigma_c$	$\rho_{bc}\sigma_a\sigma_c$	σ_c^2	…
…	…	…	…	…

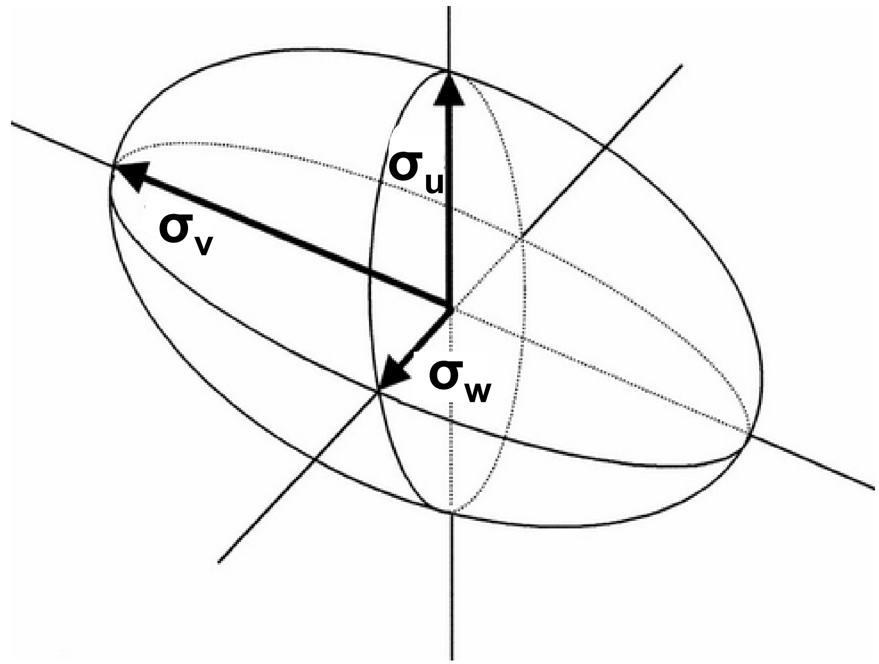
Example Covariance from CDM

- **8 x 8 matrix typical of most ASW updates**
  - Some orbit regimes not suited to solution for both drag and SRP; these covariances 7 x 7
- **Mix of different units often creates poorly conditioned matrices**
  - Condition number of matrix at right is 9.8E+11—terrible!
- **Often better numerically (and more intuitive) to separate matrix into sections**
- **First 3 x 3 portion (amber) is *position* covariance—often considered separately**

	U (m)	V (m)	W (m)	Udot (m/s)	Vdot (m/s)	Wdot (m/s)	B (m2/kg)	AGOM (m2/kg)
U	6.84E+01	-2.73E+02	6.38E+00	2.76E-01	-7.14E-02	8.75E-03	-3.83E-02	-3.83E-02
V	-2.73E+02	1.10E+05	3.23E+01	-1.17E+02	-8.99E-02	2.51E-02	-1.28E-01	-1.28E-01
W	6.38E+00	3.23E+01	4.47E+00	-3.26E-02	-6.83E-03	1.81E-03	-3.73E-03	-3.73E-03
Udot	2.76E-01	-1.17E+02	-3.26E-02	1.24E-01	1.10E-04	-2.47E-05	1.46E-04	1.46E-04
Vdot	-7.14E-02	-8.99E-02	-6.83E-03	1.10E-04	7.57E-05	-9.39E-06	4.10E-05	4.10E-05
Wdot	8.75E-03	2.51E-02	1.81E-03	-2.47E-05	-9.39E-06	2.06E-05	-4.39E-06	-4.39E-06
B	-5.07E-03	1.30E+00	4.34E-05	-1.38E-03	7.97E-07	7.26E-07	1.64E-05	-6.28E-07
AGOM	-3.83E-02	-1.28E-01	-3.73E-03	1.46E-04	4.10E-05	-4.39E-06	-6.28E-07	2.31E-05

Position Covariance Ellipse

- **Position covariance defines an "error ellipsoid"**
  - Placed at predicted satellite position
  - Square root of variance in each direction defines each semi-major axis (UVW system used here)
  - Off-diagonal terms rotate the ellipse from the nominal position shown
- **Ellipse of a certain "sigma" value contains a given percentage of the expected data points**
  - 1-σ: 19.9%
  - 2-σ: 73.9%
  - 3-σ: 97.1%
  - Note how much lower these are than the univariate normal percentage points



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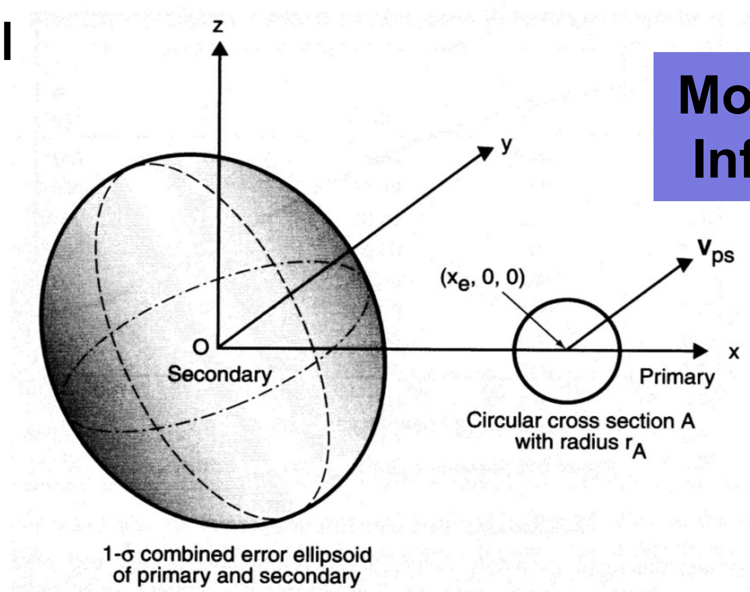
Covariance in Calculation of Probability of Collision (Pc)

- **Primary and secondary covariances combined and projected into *conjunction plane* (plane perpendicular to relative velocity vector at TCA)**
- **Primary placed on x-axis at (miss distance, 0) and represented by circle of radius equal to sum of both spacecraft circumscribing radii**
- **Z-axis perpendicular to x-axis in conjunction plane**
- **Pc is portion of combined error elli[p... hard-body radius circle**

$$P_C = \frac{1}{\sqrt{(2\pi)^2 |C^*|}} \iint_A \exp\left(-\frac{1}{2}\vec{r}^{\,T} C^{*-1} \vec{r}\right) dX dZ$$

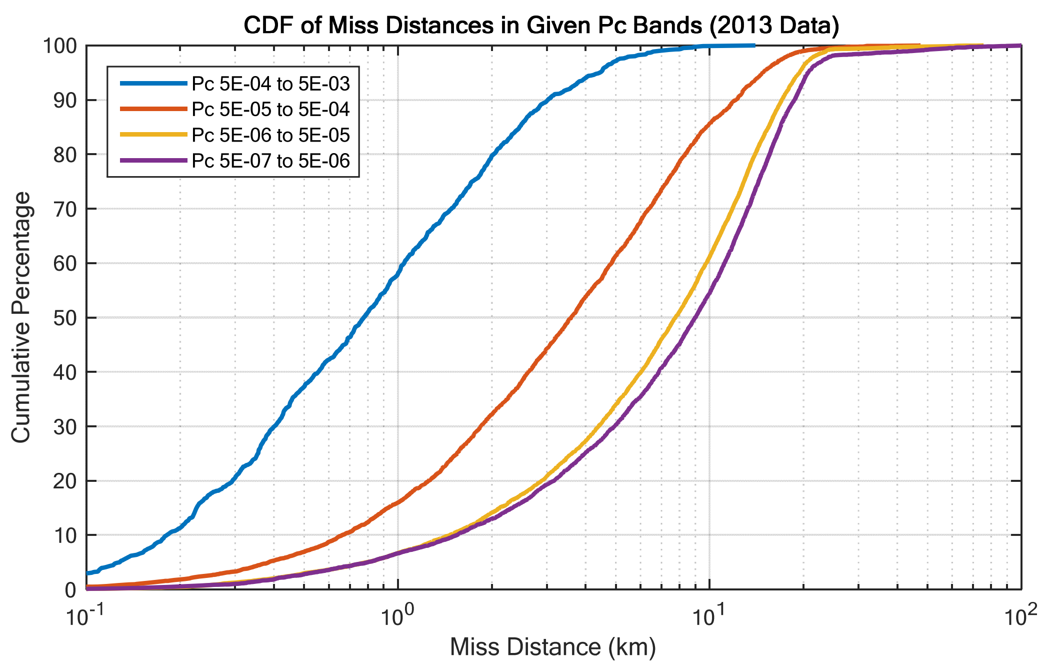
Covariance essential to Pc calculation, which is the most important factor in collision risk assessment



More Info

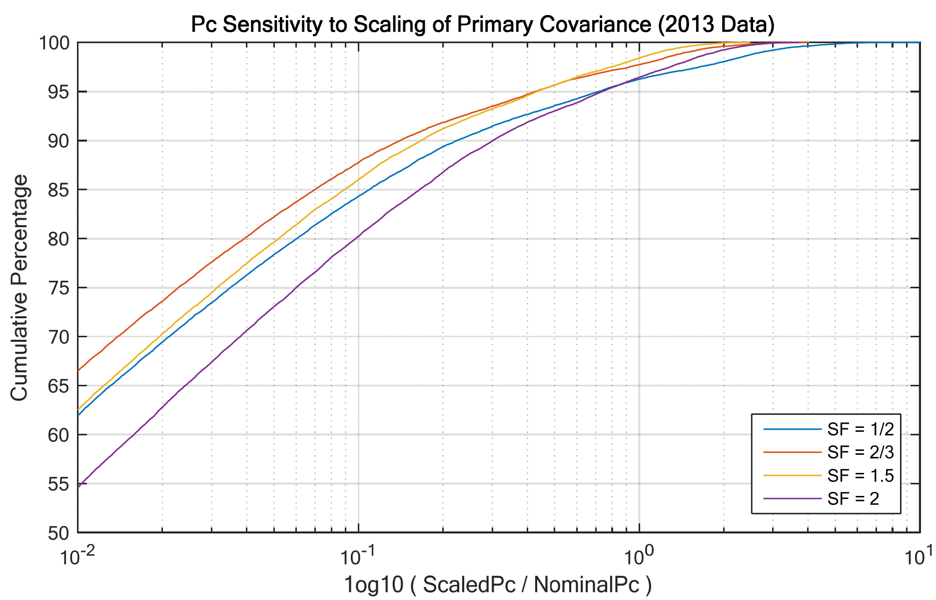
Pc vs Miss Distance Calculations

- **Pc is the best single-parameter encapsulation of the risk**
- **Without Pc, have only the miss distance**
- **Correlation between miss distance and Pc very poor**
  - Four Pc bands shown below; correlation with miss distance poor in all cases
- **Important to have Pc, and covariance necessary for its calculation**



Pc Sensitivity to Scaling of Primary Covariance

- If covariance of primary inadequately sized, Pc affected
- Graph below shows Pc differences between nominal value and recalculation with primary covariance rescaled (SF 0.5 – 2)
- ~2-5% of cases show differences greater than an order of magnitude—can affect operational conclusions
- Important to get primary covariance right



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Batch Epoch Covariance Generation (1 of 2)

- **Batch least-squares update (ASW method) uses the following minimization equation**
  - $dx = (A^TWA)^{-1}A^TWb$
    - dx is the vector of corrections to the state estimate
    - A is the time-enabled partial derivative matrix, used to map the residuals into state-space
    - W is the “weighting” matrix that provides relative weights of observation quality (usually 1/σ, where σ is the standard deviation generated by the sensor calibration process)
    - b is the vector of residuals (observations – predictions from existing state estimate)
- **Covariance is the collected term $(A^TWA)^{-1}$**
  - A the product of two partial derivative matrices:
    - $A = \frac{\partial(obs)}{\partial X_0} = \frac{\partial(obs)}{\partial X}\frac{\partial X}{\partial X_0}$
    - First term: partial derivatives of observations with respect to state at obs time
    - Second term: partial derivatives of state at obs time with respect to epoch state

Batch Epoch Covariance Generation (2 of 2)

- **Formulated this way, this covariance matrix is called an *a priori* covariance**
  - A does not contain actual residuals, only transformational partial derivatives
  - So $(A^TWA)^{-1}$ is a function only of the amount of tracking, times of tracks, and sensor calibration relative weights among those tracks
    - Not a function of the actual residuals from the correction
- **Limitations of *a priori* covariance**
  - Does not account well for unmodeled errors, such as transient atmospheric density prediction errors
    - Because not examining actual fit residuals
  - W-matrix only as good as sensor calibration process
    - Principal weakness of present process, but expected to be improved eventually with JSpOC Mission System (JMS) upgrades

Covariance Propagation Methods

- **Full Monte Carlo**
  - Perturb state at epoch (using covariance), propagate each point forward to t_n with full non-linear dynamics, and summarize distribution at t_n
- **Sigma point propagation**
  - Define small number of states to represent covariance statistically, propagate set forward by time-steps, reformulate sigma point set at each time-step, and use sigma point set at t_n to formulate covariance at t_n
- **Linear mapping**
  - Create a state-transition matrix by linearization of the dynamics and use it to propagate the covariance to t_n by pre- and post-multiplication
- **All three of above methods legitimate**
  - List moves from highest to lowest fidelity and computational intensity

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Covariance Tuning

- **For CA, position covariance needs to be a realistic representation of the state uncertainty volume at the propagation point of interest**
- **Two aspects to this requirement**
  - Does the position error volume conform to a trivariate Gaussian distribution?
  - If so, is it of the proper dimensions and orientation?
- **Regarding the first item, extensive study has confirmed that this is not an issue for high-PC events (Pc>1E-04)**
  - Ghrist and Plakalovic (2012)
  - 248 cases examined in different orbit regimes, with prop times of 2 to 7 days
  - 2-d Pc calculation compared to Monte Carlo (with 4E+07 trials)
  - Only one case of more than 10% deviation between 2-d and MC calculation
    - And 10% deviation not considered operationally significant
  - Explanation: high Pc requires covariance overlap near the centers of the covariances—a part that is not affected by non-Gaussian alterations
- **Second item is area of legitimate concern**

Covariance Tuning: Covariance Realism Evaluation Method

- **Presume reference orbit (or precision observation) available for a satellite**
- **Position differences between predicted ephemeris and precision position (from reference orbit or observation) are dU, dV, and dW**
  - Can be collected into vector ε
- **Mahalanobis distance ($\varepsilon * C^{-1} * \varepsilon^T$) represents the ratio of the difference to the covariance's prediction**
  - For a trivariate distribution, expected value is 3
- **A group of such calculations should conform to a chi-squared distribution with three degrees of freedom**
- **This method (distribution testing of groups of such calculations) used to determine if covariance properly sized**

Covariance Tuning: Covariance Irrealism Remediation

- **Examine individual component performance of covariance modeling to determine principal sources of the irrealism**
  - Deviation probably stems from non-conservative force modeling (drag and/or solar radiation pressure)
- **If using process noise, tune/modify process noise matrix to attempt to compensate**
  - Originally directed at geopotential mismodeling; but with common use of higher-order theories, no longer the principal source of errors
- **If using batch methods, include consider parameters**
  - Additive value applied to either the drag or solar radiation pressure variances (or both) in order to make them larger
    - Poor modeling of these phenomena requires larger uncertainty estimate
  - Through cross-correlation terms, these variances will affect the other covariance parameters through the linear state transition
- **Continue tuning process until proper distribution of calculated Mahalanobis distances achieved**

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Covariance Theory Compatibility

- **Batch covariance is governed by the amount and quality of tracking data used in the OD**
- **Propagated covariance is a product of the particular propagation technique used and tuning applied**
  - Tuning itself a function of the adequacy of the OD force modeling
- **Thus, important that covariance be generated from same OD basis that produced the state estimate**
- **This is not possible for O/O ephemerides that lack a covariance**
  - Forced to use O/O state estimate and ASW covariance (or, worse, a synthesized covariance when no ASW covariance exists)
  - Such a covariance a questionable representation of O/O ephemeris error
    - $\sigma_{O/O}^2 = \sigma_{ASW}^2 + \sigma_{Diff}^2$
    - The difference variance is unknown, so using an ASW covariance with an O/O ephemeris understates the uncertainty but by an unknown amount

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CARA O/O Covariance Needs

- **O/O ephemerides need to contain accompanying covariances**
  - Especially true for ephemerides that contain planned maneuvers
    - No parallel ASW solution from which to "borrow" covariance for primary
  - One covariance entry for each ephemeris point is standard
    - Could possibly accommodate less frequent spacing, but would not conform to CCSDS standard and probably more difficult than the default approach
  - Full covariance (8 x 8) preferred; 3 x 3 (position covariance) usable with certain assumptions
- **Delivery of covariance can form basis for including maneuver execution error in maneuver trade-space analyses**
  - Open area for collaborative analysis with O/Os
- **Daily delivery of ephemerides desirable**
  - Propagation error can effect large changes in the Pc
  - This error minimized for both states and covariances through daily updates
    - Propagation time to TCA reduced

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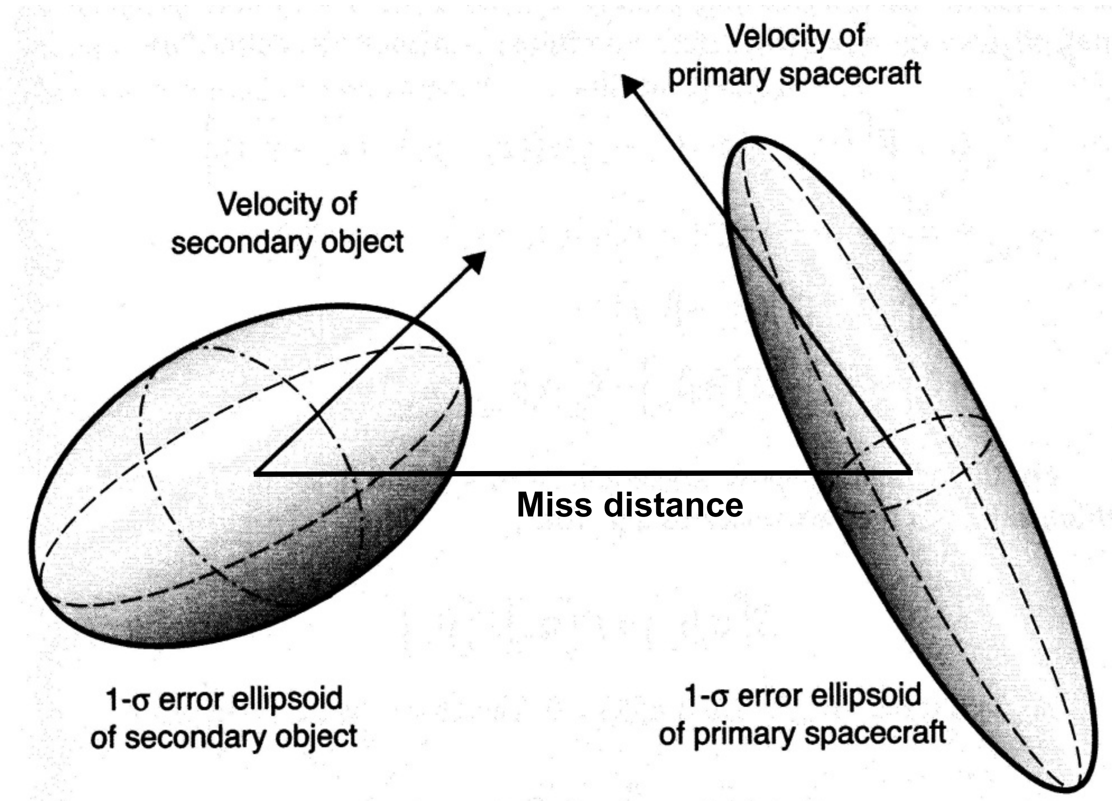
Conclusions

- **Properly tuned O/O covariance very important to CA**
  - Incorporation into daily deliveries of O/O ephemerides highly desirable
- **Covariance theory compatibility very important**
  - Applaud recent efforts (*e.g.,* ESMO) to develop covariance generation capabilities
- **Variety of methods for covariance production, propagation, and tuning**
  - CARA ready to assist with advice for production and tuning implementation
- **Can incorporate O/O covariances into CARA operational software and processes as soon as such products are ready**

BACK-UP SLIDES

Calculating Probability of Collision (Pc): Situation at Time of Closest Approach (TCA)

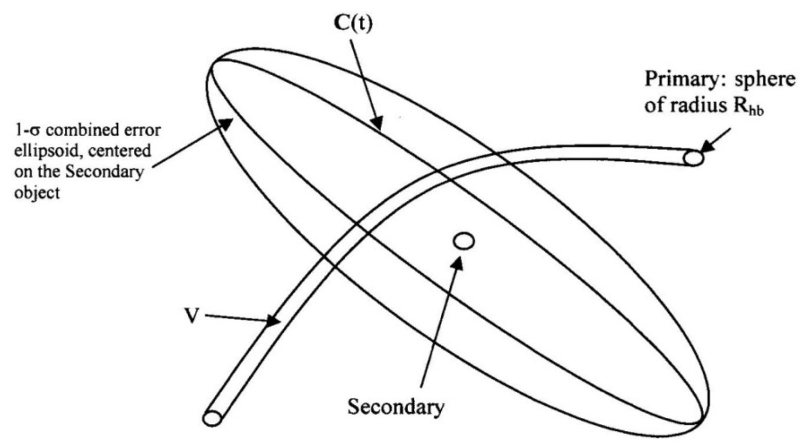


Calculating Pc: 2-D Approximation (1 of 3)
Relative Position Covariance

- **Assumptions**
  - Covariances of primary and secondary objects are uncorrelated
- **Result**
  - All of the relative position error can be centered at one of the two satellite positions
    - Position of the secondary is typically used
  - Relative position error can be expressed as the additive combination of the two position covariances (proof given in Chan 2008)
    - $C_a + C_b = C_c$
    - Both covariances must be transformed into a common coordinate frame before combination

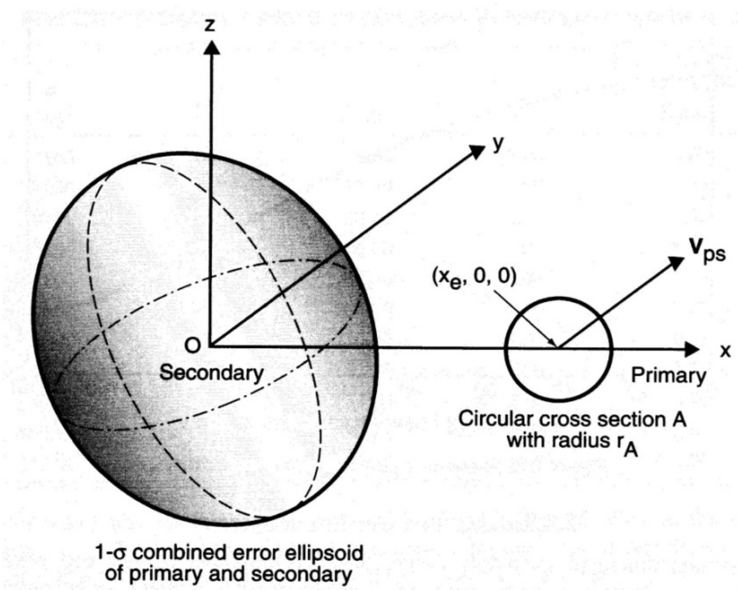
Calculating Pc: 2-D Approximation (2 of 3) Projection to Conjunction Plane

- **Combined covariance centered at position of secondary at TCA**
- **Primary path shown as curved "soda straw"**
- **If conjunction duration is very short**
  - Motion can be considered to be rectilinear—soda straw is straight
  - Conjunction will take place in 2-d plane normal to the relative velocity vector and containing the secondary position
  - Problem can thus be reduced in dimensionality from 3 to 2
- **Need to project covariance and primary path into "conjunction plane"**



Calculating Pc: 2-D Approximation (3 of 3) Conjunction Plane Construction

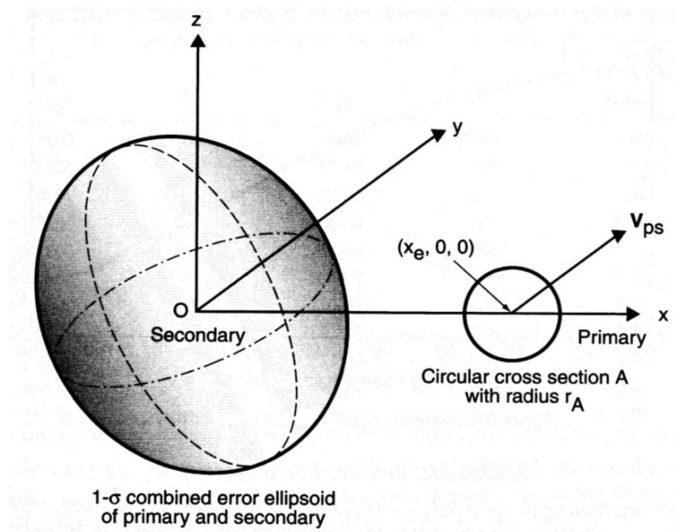
- **Combined covariance projected into plane normal to the relative velocity vector and placed at origin**
- **Primary placed on x-axis at (miss distance, 0) and represented by circle of radius equal to sum of both spacecraft circumscribing radii**
- **Z-axis perpendicular to x-axis in conjunction plane**



Probability of Collision Computation

- **Pc is the portion of the density that falls within the HBR circle (r is [x z] and C* is the projected covariance)**

$$P_C = \frac{1}{\sqrt{(2\pi)^2 \left|C^*\right|}} \iint_A \exp\left(-\frac{1}{2}\vec{r}^{\,T} C^{*-1} \vec{r}\right) dXdZ$$



Conclusion: covariance essential to Pc calculation, which is the most important factor in collision risk assessment

Return

Covariance Propagation
Method 1: Full Monte Carlo (1 of 2)

- **Creates *n* state (position and velocity) perturbations at epoch**
  - Covariance at epoch describes uncertainty of state at epoch
  - Can use this to create set of *n* possible realizations of the epoch state, conforming to the distribution parameters specified by the covariance
- **Propagates each of these forward to the time of interest**
  - Use the full non-linear dynamics of the propagator
  - Thus produce *n* states at TCA (for CA application)
- **Summarizes set of n states statistically**
  - Usually empirically, through non-parametric techniques (*e.g.,* percentiles, empirical distribution functions)

Covariance Propagation
Method 1: Full Monte Carlo (2 of 2)

- **Advantages**
  - Most accurate method of characterizing uncertainty, as there are no inherent simplifying assumptions or activities (such as linearization)
- **Disadvantages**
  - Very large number of samples required to characterize tails of distribution
  - Far more computationally intensive than other methods

Covariance Propagation
Method 2: Sigma Point Propagation (1 of 2)

- **Usually applied to unscented Kalman filter (UKF) OD processes**
- **Generates a (relatively small) set of sample states (called *sigma points*) about the nominal state, which represent the uncertainty**
  - Sample covariance of sigma points should approximate covariance from state estimate
  - Theory says 2L+1 sigma points needed, where L is state degrees of freedom
    - Can increase this somewhat if prior information available; will improve accuracy of uncertainty volume reconstruction
- **Propagates sigma points to next time step**
- **Constructs covariance (and state) at this future state from sigma points**
  - Weighting functions often assembled to assist in reconstruction

Covariance Propagation
Method 2: Sigma Point Propagation (2 of 2)

- **Advantages**
  - Greatly reduces number of non-linear propagations
    - However, has to perform sigma-point construction at each time-step
- **Disadvantages**
  - Makes (and imposes) *a priori* determination of future uncertainty volume distribution
  - Still requires multiple non-linear propagations

Covariance Propagation
Method 3: Linear Mapping (1 of 2)

- **Non-linear dynamics of orbit propagation can be linearized**
  - These linear approximations valid for “short” periods about epoch state
- **State transition matrix (Φ) the encapsulation of this linearization**
  - Can be used for state propagations (but often is not)
  - Can also be used for propagation of covariance $[\Phi(t,t_o)*C(t_o)*\Phi^T(t,t_o)]$
- **Covariance propagation can also be augmented via the addition of “process noise”**
  - Process noise matrix (Q) formulated, which specifies acceleration uncertainty in each coordinate principal direction
    - Intent is to compensate for unmodeled and inadequately-modeled perturbations
    - Can potentially remediate some of the limitations introduced by the linearization
  - Process noise matrix propagated through use of a process noise transition matrix, in a manner similar to state transition: $[\Gamma(t,t_o)*Q(t_o)*\Gamma^T(t,t_o)]$

Covariance Propagation
Method 3: Linear Mapping (2 of 2)

- **Advantages**
  - Much faster and less computationally intensive than other methods
  - Process noise provides mechanism for covariance tuning/realism adjustments
- **Disadvantages**
  - Least accurate, especially for long propagations
  - Imposes *a priori* statistical structure upon uncertainty volume
  - Use of process noise requires careful tuning process

Return